Written Exam at the Department of Economics winter 2019-20

Economics of the Environment, Natural Resources and Climate Change

Final Exam

9 January 2020

(3-hour closed book exam)

Answers only in English.

This exam question consists of 4 pages in total

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- leave the examination.
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- Or if you otherwise violate the rules that apply to the exam

Exercise 1. Climate Policy and The Green Paradox (indicative weight: 3/4)

The model considered in this exercise uses the following notation:

t	= Time index.
V_t	= Net present value of profits.
R_t	= Extraction intensity of the fossil fuel (flow).
R_t^d	= Demand for the fossil fuel (flow).
P_t	= Real producer price of the fossil fuel.
τ_t	= Per unit carbon tax.
$P_t + \tau_t$	= Real consumer price of the fossil fuel.
τ_t / P_t	= Ad valorem carbon tax.
θ_t	= Tax wedge.
r	= Real interest rate (assumed exogenous and constant over time).
S_t	= Stock of fossil fuel reserves.
D_t	= Present cost of damages from climate change.
x_t	= Social damage cost of consuming fossil fuels.

Consuming/burning one unit of the fossil fuel results in one unit of carbon emission.

A representative mining firm extracts a fossil fuel resource and sells it on the market. This mining firm maximizes the present value of its profits given by (1) subject to the resource constraint (2).

$$V_0 = \int_0^\infty P_t R_t \mathrm{e}^{-rt} \, \mathrm{d}t, \quad r > 0. \tag{1}$$

$$S_t = S_0 - \int_0^t R_v \, \mathrm{d}v. \tag{2}$$

The initial stock of fossil fuel reserves, $S_0 > 0$, is exogenous, and it can be shown that (2) implies that:

$$\dot{S}_t \equiv \frac{\mathrm{d}S_t}{\mathrm{d}t} = -R_t.$$

The representative mining firm treats the fossil fuel price path as exogenous, and for simplicity, the real interest rate is exogenous in this model.

Time is continuous and $t \geq 0$.

Question 1.1

Set up the current value Hamiltonian for the representative mining firm's problem and derive the first-order conditions for its solution, including the transversality condition. Denote the shadow price of the resource stock by λ_t. Give a brief interpretation of the transversality condition.

Question 1.2

Use the first-order conditions to derive the Hotelling rule. Explain the intuition behind your result.

Question 1.3

The demand for the fossil fuel resource is given by:

$$R_t^d = (P_t + \tau_t)^{-\gamma}, \quad \gamma > 0,$$

where τ_t is a per unit carbon tax.

Define the tax wedge $\theta_t \equiv 1 + \tau_t / P_t$. Multiplying the producer price, P_t, by the tax wedge equals the consumer price, $\theta_t P_t$. Assume that the wedge evolves according to:

$$\theta_t = \theta_0 \mathrm{e}^{gt}, \quad g > 0, \quad \theta_0 > 1 \text{ given.}$$

The above equation implies that the ad valorem tax, τ_t / P_t, increases over time. Accordingly, the wedge between the consumer and producer price increases over time.

Show that the extraction rate is given by:

$$R_t = S_0 \gamma (r + g) \mathrm{e}^{-\gamma (r+g) t}.$$

Hint: you need to employ the demand function, the Hotelling rule, the expression for θ_t, as *well as the equilibrium condition:*

$$S_0 = \int_0^\infty R_t \, \mathrm{d}t.$$

It may be useful to start by showing that: $R_t^d = (P_0\theta_0)^{-\gamma} \mathrm{e}^{-\gamma(r+g)t}$.

Question 1.4

As mentioned above, the flow of carbon emissions equals the consumption flow of fossil fuels, R_t^d.

Show what happens to short-run carbon emissions if the government at time $t = 0$ increases the growth rate of the tax wedge (an increase in g)? Explain your finding intuitively.

Question 1.5

Let the present cost of damages from climate change be given by:

$$D_0 = \int_0^\infty x_t R_t^d \mathrm{e}^{-\rho t} \, \mathrm{d}t, \quad \rho > 0, \tag{3}$$

where x_t is the social damage cost of consuming/burning a unit of fossil fuel at time t. Let this damage cost evolve according to:

$$x_t = x_0 e^{\delta t}, \quad 0 < \delta < \rho, \quad x_0 > 0 \text{ given.} \tag{4}$$

Show that the present cost of damages can be expressed as:

$$D_0 = (r+g)\frac{\gamma x_0 S_0}{\rho - \delta + \gamma(r+g)}. \tag{5}$$

Question 1.6

Show what happens to the present cost of damages if the government at time $t = 0$ increases the growth rate of the tax wedge (an increase in g)? Explain your finding intuitively.

Question 1.7

The government would like to tighten its climate policy. Specifically, the government would like to reduce short-run emissions. It considers the following two alternative policies:

- **Policy 1:** A constant tax wedge: $\theta_t = \bar{\theta} > 1$.

- **Policy 2:** A decreasing tax wedge with the same initial tax wedge as in Policy 1: $\theta_t = \bar{\theta}e^{\phi t}$, where $\phi < 0$.

Determine the preferred policy given the ambition of the government. Explain the intuition behind your findings.

Question 1.8

This exercise has focused on carbon taxes, but there exists many other climate policies.

Give some examples of climate policies that are not vulnerable to the mechanisms highlighted by the Green Paradox hypothesis. Briefly explain the intuition.

Exercise 2: Genuine Saving and Sustainability (indicative weight: 1/4)

(Hint: You may provide purely verbal answers to the questions in this exercise, but you are also welcome to include equations if you find it useful)

Question 2.1

Explain (briefly) the concept "genuine saving".

Question 2.2

Discuss how the concepts "genuine saving" and "sustainable development" are connected.